

Shear Strength of Reinforced Concrete Beams per ACI 318-02

Course Content

1. Introduction

In a simple beam subjected to bending, the fibers above the neutral axis are in compression, whereas tensile stresses occur in the fibers below this axis. The factors influencing shear strength and formation of inclined cracks are so numerous and complex that a definitive conclusion regarding the exact mechanism of inclined cracking resulting from high shear is difficult to establish. The behavior of reinforced concrete beams at failure in shear is distinctly different from their behavior in flexure. They fail abruptly without sufficient advanced warning, and the diagonal cracks that develop are considerably larger than the flexural cracks.

A concrete beam reinforced with longitudinal steel only, will develop diagonal tensile stresses that tend to produce cracks, as indicated in Fig. 1. These cracks are vertical at the center of span and become more inclined as they approach the supports, sloping at an angle approximately 45° . The stresses that cause these cracks are known as diagonal tension. In order to prevent this type of failure additional reinforcing bars are added, which are called stirrups (perpendicular to the longitudinal axis of beam).

A practical assumption is that the intensity of the vertical shear is considered to be a measure of the intensity of the diagonal tension. Thus, when we use the term shear, we really refer to the diagonal tension stresses. Since the strength of concrete in tension is significantly lower than its strength in compression, design for shear is of major import in concrete structures.

This course will use the *strength design method or ultimate strength design (USD)* to determine the shear strength of nonprestressed concrete beams. This method is consistent with the prevalent methodology endorsed by ACI 318-02.

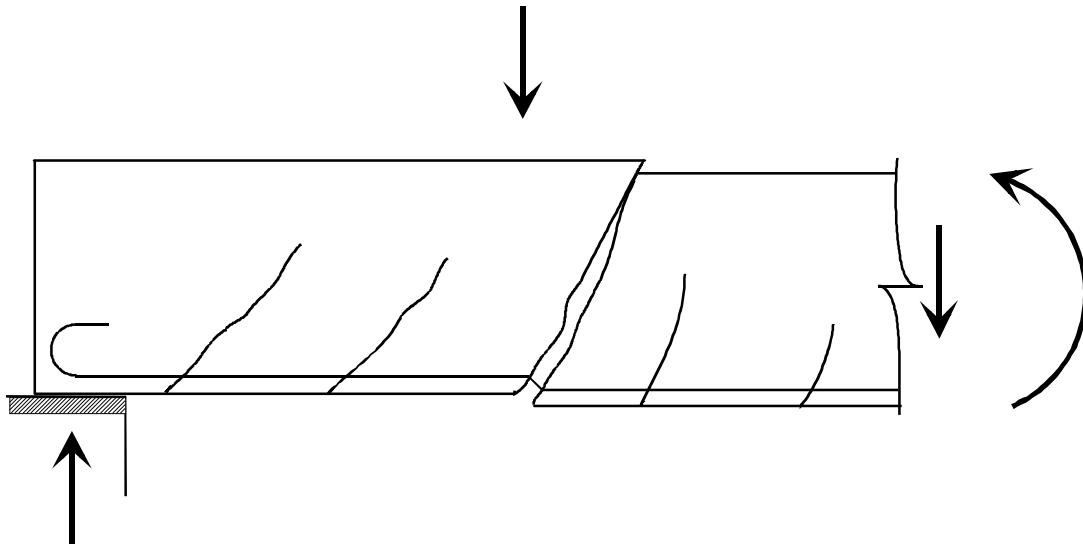


Figure No. 1

2. Shear in Homogeneous Elastic Beams

Homogeneous elastic beams have stresses proportional to strains, and the shear stresses are obtained from the principles of classical mechanics of materials, such as equation (1):

$$v = \frac{V Q}{I b} \quad (\text{Eq. 1})$$

V = Shear force at section under consideration

Q = statical moment about neutral axis of that portion of cross section lying between a line through point in question parallel to neutral axis and nearest face (upper or lower) of beam.

I = moment of inertia of cross section about neutral axis

b = width of beam at given point

The infinitesimal elements A_1 and A_2 of a rectangular beam in Figure 2 are shown with the tensile normal stress f_t and shear stress v across plane $a_1 - a_1$, and $a_2 - a_2$ at a distance y from the neutral axis.

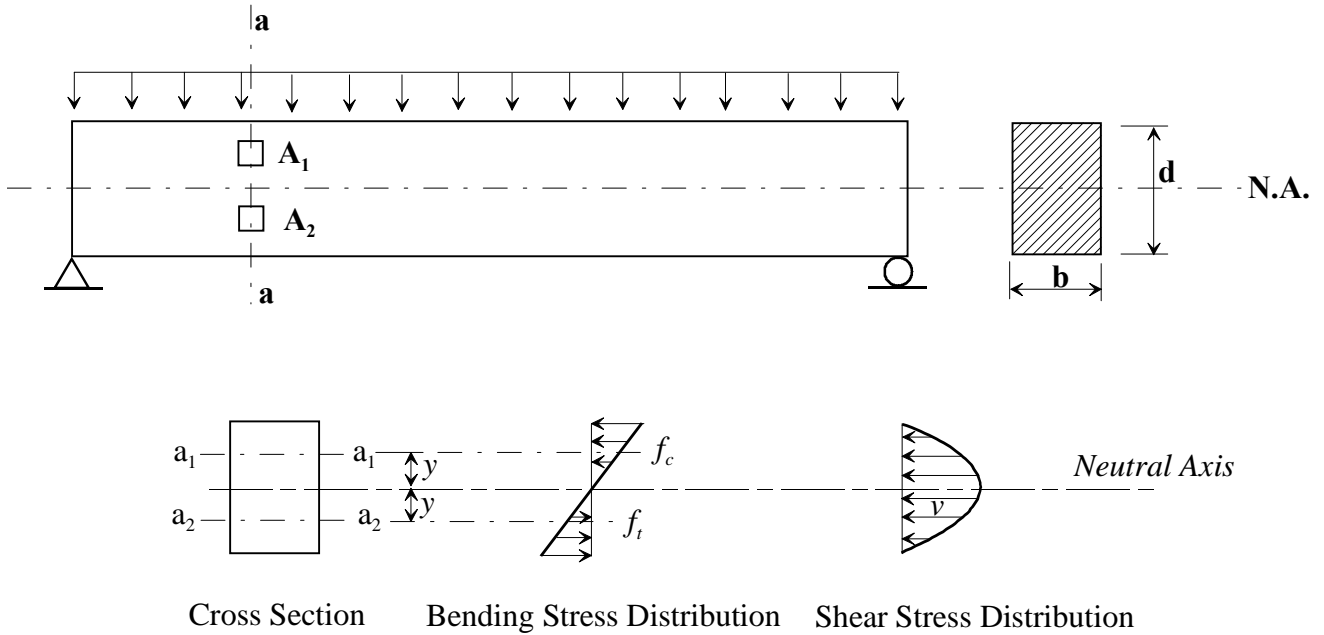


Figure No. 2

Figure 3 shows the internal stresses acting on the infinitesimal element A_2 , and using Mohr's circle, the principal stresses for element A_2 in the tensile zone below the neutral axis can be computed as:

$$f_{t(max)} = \frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + v^2} \quad \text{principal tension} \quad \text{(Eq. 2.a)}$$

$$f_{c(max)} = \frac{f_t}{2} - \sqrt{\left(\frac{f_t}{2}\right)^2 + v^2} \quad \text{principal compression} \quad \text{(Eq. 2.b)}$$

The same procedure can be used to find the internal stresses on the infinitesimal element A_1 .

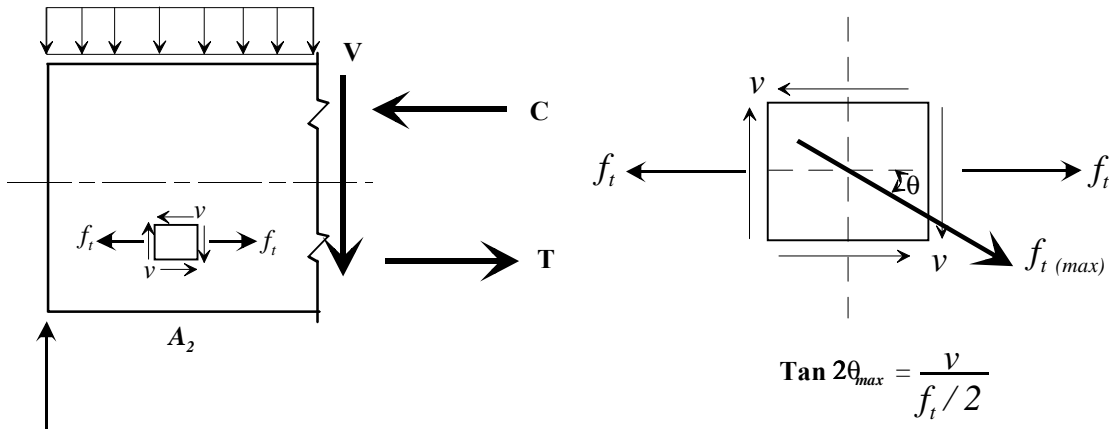


Figure No. 3

Stress state in element A_2

The behavior of reinforced concrete beams is more complex since the tensile strength of the concrete is approximately 10% of its compressive strength. For compressive element A_1 the maximum principal stress is in compression, therefore cracking is prevented. On the other hand, for element A_2 below the neutral axis the maximum principal stress is in tension, hence cracking may be induced. For an element at the neutral axis near the support, the state of stress would be that of pure shear thus having the maximum tensile and compressive principal stress acting at a 45° angle from the plane of the neutral axis.

Due to the complex mechanism of shear resistant mechanism in reinforced concrete beams (nonhomogeneous, non-elastic behavior), the ACI –ASCE Joint Committee 426 gives an empirical correlation of the basic concepts developed from extensive test results.

3. Modes of Failure of Concrete Beams without Shear Reinforcement

In areas where the bending moments are large, cracks develop almost perpendicular to the axis of the beam. These cracks are termed *flexural cracks*. In region of high shear due to the diagonal tension, the cracks develop as an extension of the flexural crack, and they are called *flexure shear cracks*. Figure No. 4 shows the types of cracks expected in a

reinforced concrete beam with or without effective diagonal tension reinforcement.

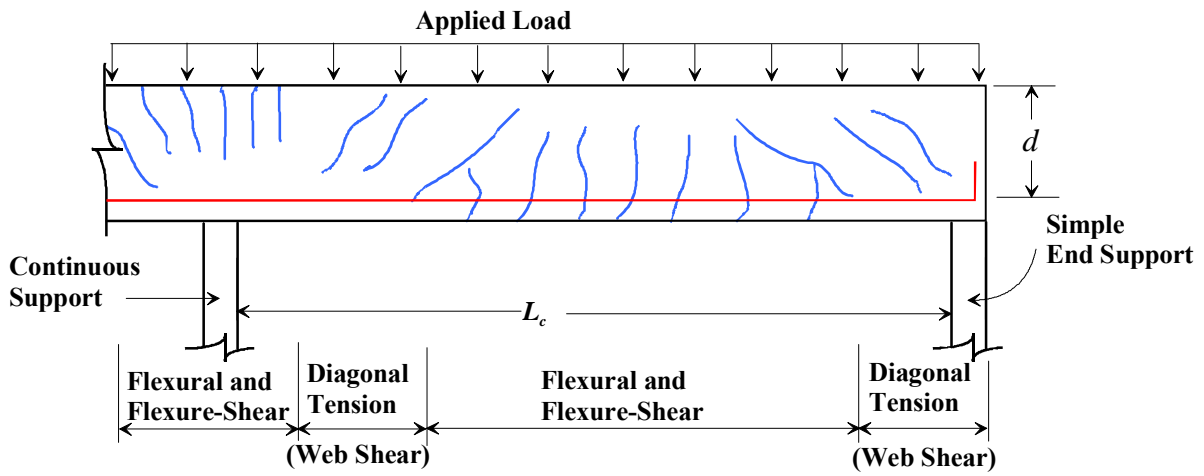


Figure No. 4

In the region of flexural failure, the cracks are mainly vertical in the middle third of the beam span and perpendicular to the lines of principal stresses. The dominant cause of these cracks is large flexural stress f .

Diagonal tension cracks may precipitate the failure of the beam if the strength of the beam in diagonal tension is lower than its strength in flexural tension. This type of crack starts with the development of a few vertical flexural at midspan. Thereafter, the bond between the reinforcing steel and the surrounding concrete is destroyed at the support. Finally, two or three diagonal cracks develop at about $1\frac{1}{2}$ to $2d$ distance from the face of the support. At this stage it is recognized that the beam has reached its shear strength. The beams categorized as intermediate beam usually fall in this category.

The shear strength of deep beams is predominantly controlled by the effect of shear stress. These beams have a small shear span/depth ratio, a/d and are not part of the scope of this work. ACI 318 section 11.8 addresses the shear strength of deep beams. See Fig. 5 and Table No. 1 for classification of beams as a function of beam slenderness.

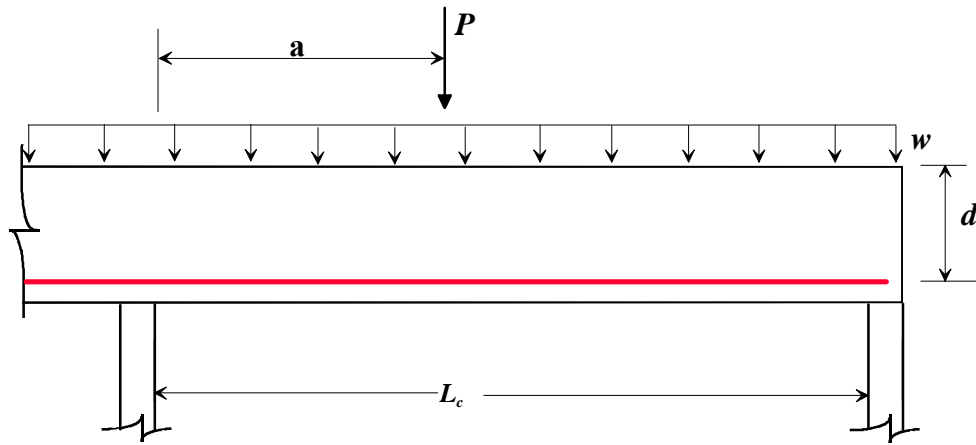


Figure No. 5

TABLE 1: Beam Slenderness Effect on Mode of Failure

Beam Category	Failure Mode	Shear Span/Depth Ratio as a Measure of Slenderness	
		Concentrated Load, a/d	Distributed Load, L_c/d
Slender	Flexure	Exceeds 5.5	Exceeds 16
Intermediate	Diagonal Tension	2.5-5.5	11-16
Deep	Shear Compression	1-2.5	1-5

For a uniformly distributed load, a transition develops from deep beam to intermediate beam effect.

4. Shear Strength Criteria

The ACI provisions of Chapter 11 considers the shear strength of beams on an average shear stress on the full effective cross section $b_w d$. In a member without shear reinforcement, shear is assumed to be resisted by the concrete web. In a member with shear reinforcement, the shear strength of the beam is considered to be the sum of shear strength provided by the concrete and that attributable to the shear reinforcement.

Except for members designed in accordance with ACI 318 Appendix A (Strut-and-Tie Models), the strength of a cross section shall be taken as:

$$\phi V_n \geq V_u \quad \text{(Equation 3)}$$

Where, V_u is the factored shear force at the section considered and V_n is the nominal shear strength computed as:

$$V_n = V_c + V_s \quad \text{(Equation 4)}$$

V_s can be expressed as:

$$V_s = V_n - V_c = (V_u / \phi) - V_c \quad \text{(Equation 4a)}$$

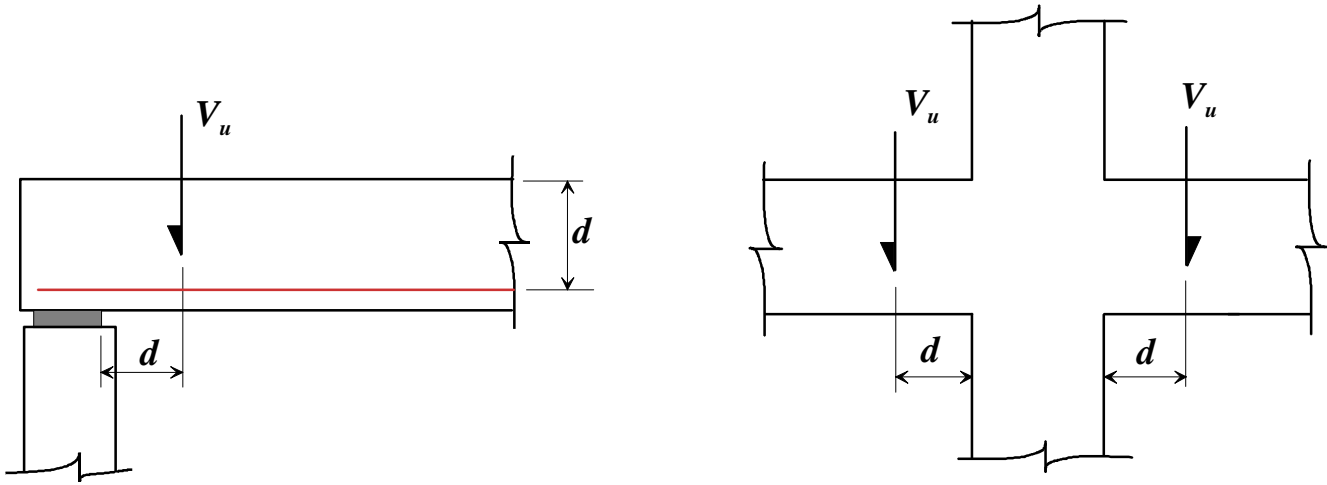
Where V_c is the strength provided by the effective concrete section and V_s is the strength provided by the shear reinforcement.

The strength reduction factor ϕ is 0.75 (ACI 318 section 9.3), a change from previous code revisions of 0.85. In computing V_c , the effect of axial tension or compression will not be considered for this course.

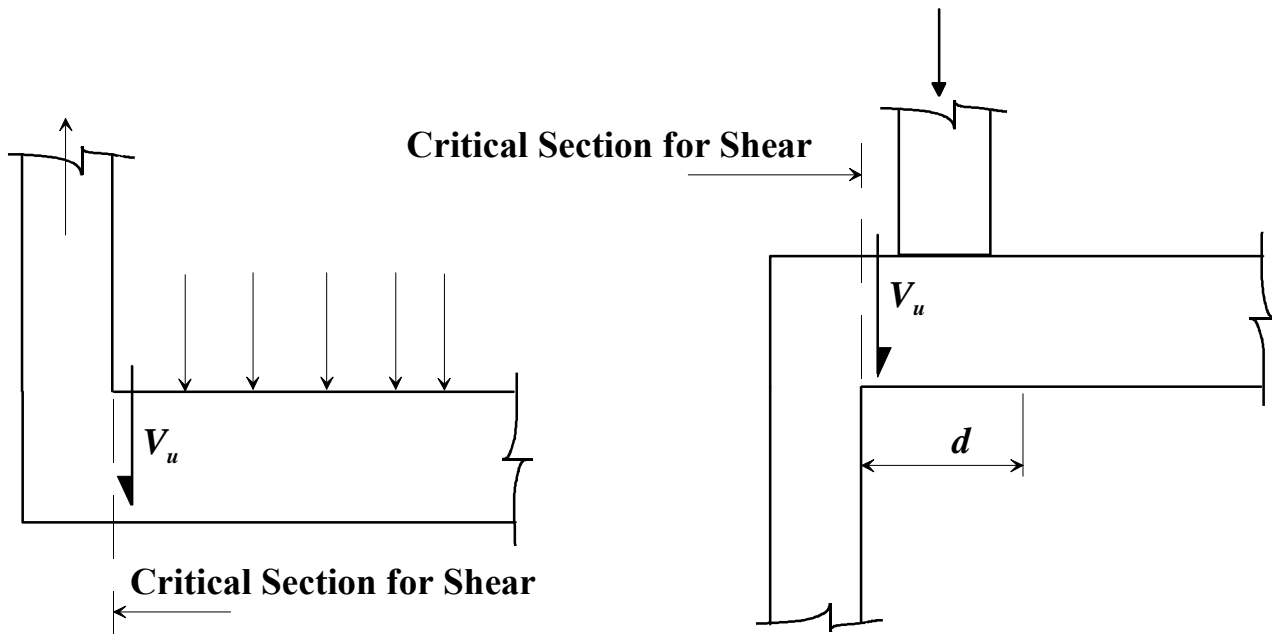
The factored shear for nonprestressed members can be designed at a location d from the face of the support for beams meeting the following criteria (See Fig. 6):

- a) Members supported by bearing at the bottom of the member
- b) Members framing monolithically into another member

In both of these cases the applied loads are located at the top of the member under consideration.



Typical support conditions where the shear force at a distance d from the support may be used



Typical support conditions where the provision distance d from the support does not applied

Figure No. 6

5. Shear Strength provided by Concrete for Nonprestressed Members

For members subject to shear and flexure only the following equation will calculate the strength provided by the concrete:

The nominal shear strength provided by the concrete V_c is computed as:

$$V_c = 2 \sqrt{f'_c} b_w d \quad \text{(Equation 5)}$$

The value of $\sqrt{f'_c}$ in equation 5 is limited to a maximum of 100 psi.

ACI 318 makes exception to this requirement for beams meeting the minimum web reinforcement expressed in equation 9 under section 6 of this course (see ACI section 11.1.2.1 for more details).

This shear strength may also be computed by the more detailed calculation for members subject to shear and flexure only:

$$V_c = \left[1.9 \sqrt{f'_c} + 2500 \rho_w \frac{V_u d}{M_u} \right] b_w d \quad \text{(Equation 6)}$$

Where:
$$\rho_w = \frac{A_s}{b_w d}$$

The quantity computed by equation 6 should be not greater than

$$V_c = 3.5 \sqrt{f'_c} b_w d \quad \text{(Equation 7)}$$

Furthermore, $V_u d / M_u$ should not be taken greater than 1.0, and M_u is the factored moment occurring simultaneously with V_u at the section under evaluation.

For circular members, the area of concrete to be used for computing V_c could be taken as:

$$A_c = 0.8 D^2 \quad \text{(Equation 8)}$$

Where, D = Diameter of the section

Special provisions for shear strength applies to lightweight concrete, such as $f_{ct}/6.7$ is to substitute the value of: $\sqrt{f'_c}$

for computing the shear strength V_c given in equations 5 and 6

But $f_{ct}/6.7$ shall not exceed $\sqrt{f'_c}$

f_{ct} = Average splitting tensile strength of lightweight aggregate concrete, psi

6. Shear Strength provided by Shear Reinforcement

Web reinforcement needs to be provided to prevent failure due to diagonal tension. If reinforcement along the line of the tensile trajectories is provided, no shear failure can occur. Due to practical considerations other forms of reinforcing are utilized to neutralize the principal tensile stresses at the critical shear failure planes.

Vertical stirrups are the most common form of web reinforcement. Inclined stirrups, perpendicular to potential cracks may be used provided that particular care is taken to locate them in their proper location. Many engineers, however, prefer to consider vertical stirrups only (Fig. 7). Vertical stirrups are the most commonly used because they are easier to install, thus more economical, and reduce the potential of misplacement.

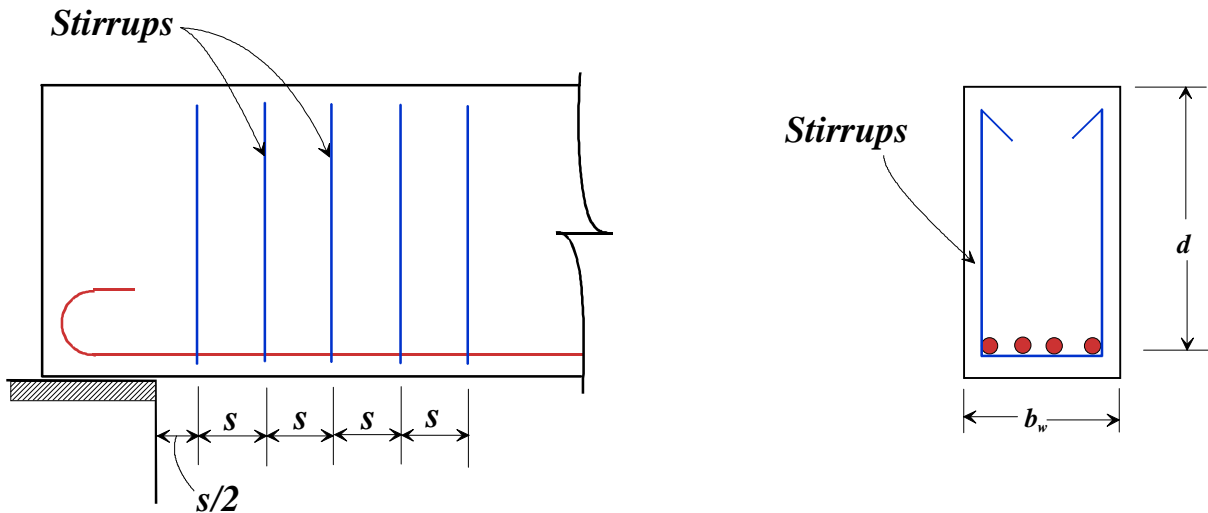


Figure No. 7

The shear reinforcement may consist of stirrups perpendicular to axis of member, welded wire fabric with wires perpendicular to axis of member, spirals, circular ties, or hoops.

The design shear strength of shear reinforcement must not exceed 60,000 psi, except that for welded deformed wire fabric the design yield strength should not exceed 80,000 psi.

Stirrups and other shear reinforcement must extend to a distance d from the extreme compression fiber and shall be adequately anchored at both ends according to ACI 318 section 12.13 to develop its design yield strength.

Where shear reinforcement is required, the minimum area of shear reinforcement required is computed by:

$$A_v = 0.75 \sqrt{f'_c} \frac{b_w s}{f_y} \geq \frac{50 b_w s}{f_y} \quad (\text{Equation 9})$$

Where b_w and s are inches, and A_v = area of shear reinforcement, all the vertical stirrup legs in the cross-section, and s = spacing of stirrups

Where the factored shear V_u exceeds the concrete shear strength ϕV_c , the shear strength to be provided by the shear reinforcement is obtained from:

$$V_s = \frac{A_v f_y d}{s} \quad (\text{Equation 10})$$

A

The shear strength V_s computed by equation 10 must not exceed

$$V_s \leq 8 \sqrt{f'_c} b_w d \quad (\text{Equation 11})$$

7. Spacing Limitations for Shear Reinforcement

Spacing of shear reinforcement placed perpendicular to axis of members (stirrups) must be less or equal to $d/2$ for nonprestressed members, nor 24 inches. This requirement ensures that any potential diagonal crack that may develop will be intercepted by a vertical stirrup, see Figure No. 8.

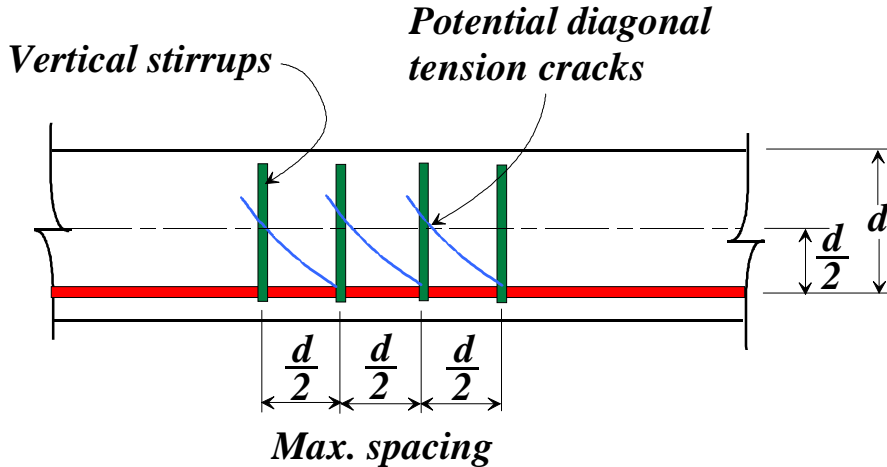


Figure No. 8

When:

$$V_s \geq 4 \sqrt{f'_c} b_w d$$

The maximum spacing given above is reduced by one half, or $S_{max} = d/4$

8. Minimum Shear Reinforcement

When the factored shear force V_u exceeds one-half the shear strength ϕV_c of the plain concrete web, a minimum amount of web steel area should be provided to prevent brittle failure by enabling both the stirrups and the beam compression zone to continue resisting the increasing shear after the formation of the first inclined crack.

The minimum web reinforcement should be computed using equation 9 shown in section 6 above.

This provision is not required for the followings, see Fig. No. 9:

- Slabs and footings
- Concrete joist construction defined in ACI 318 section 8.11
- Beams with total depth not greater than 10 in., 2.5 times thickness of flange, or 0.5 the width of web, whichever is greatest

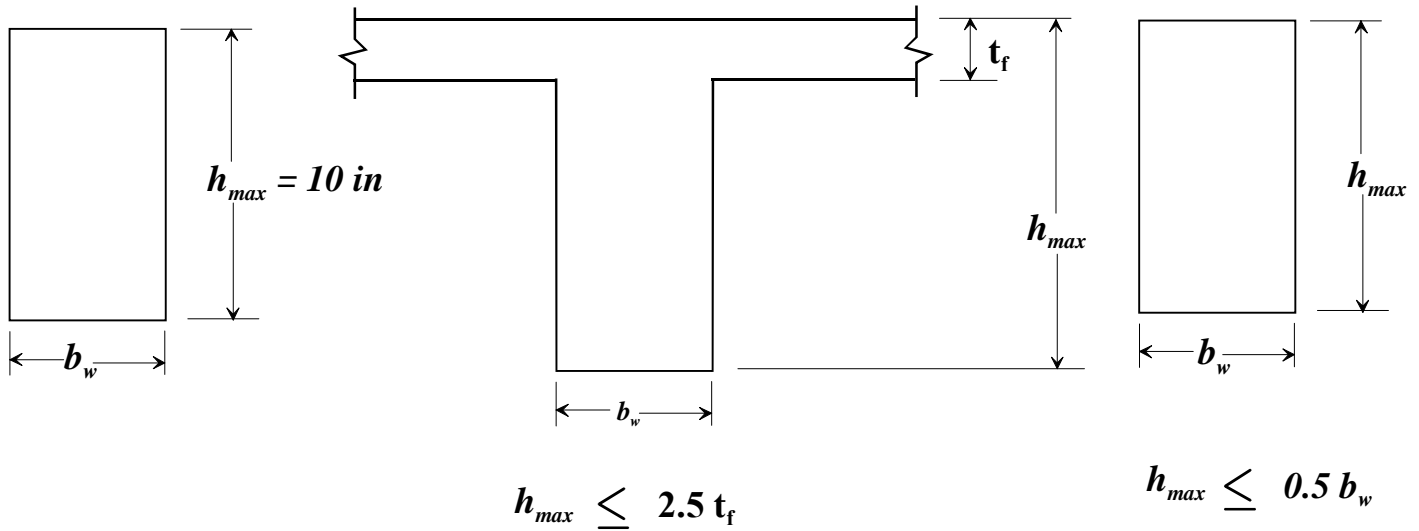


Figure No. 9

Beams not requiring web reinforcement when V_u exceeds half of the shear strength of the concrete

When the computed shear strength:

$$V_s \geq 8 \sqrt{f'_c} b_w d$$

The section is too small to ensure ductile failure, and thus it will need to be enlarged. See Table 2 for a summary of size and spacing limitations.

TABLE 2: Limitations on Size and Spacing of Stirrups

-
- | | | |
|---|------|------------------------------------|
| 1. $V_s = V_n - V_c \leq 4 \sqrt{f'_c} b_w d$ | then | $S_{max} = d/2 \leq 24 \text{ in}$ |
| 2. $V_s = V_n - V_c \geq 4 \sqrt{f'_c} b_w d$ | then | $S_{max} = d/4 \leq 12 \text{ in}$ |
| 3. $V_s = V_n - V_c > 8 \sqrt{f'_c} b_w d$ | then | Enlarge Section |
-

9. New Load Factors

ACI 318-02 has revised the load factor combinations and strength reduction factors of the 1999 code. The old factors have been kept as an alternative and are found in Appendix “C”.

The 1999 combinations have been replaced with those of ASCE 7-98. The basic formulation for the USD method is stated as:

$$\text{Design Strength} \geq \text{Required Strength}$$

$$\phi (\text{Nominal Strength}) \geq U$$

Where:

ϕ = strength reduction factor (ACI section 9.3)

U = required strength to resist factored loads or related internal moments and forces

The new ϕ factors are listed in section 9.3.2 and for shear the new factor is taken as 0.75

The load factors were changed with the goal to unify the design profession on one set of load factors and combinations, facilitating the proportioning of concrete building structures that include members of materials other than concrete (i.e. steel).

The required strength U is given as a set of load combinations in section 9.2, three of these commonly used combinations are listed below with the new load factors:

$$U = 1.2 D + 1.6 L + 0.5 (L_r + S)$$

$$U = 1.2 D + 1.6 (L_r \text{ or } S) + 1.0 L$$

$$U = 1.2 D + 1.6 W + 1.0 L + 0.5 (L_r \text{ or } S)$$

Where:

D = dead loads

L = live loads

L_r = roof live loads

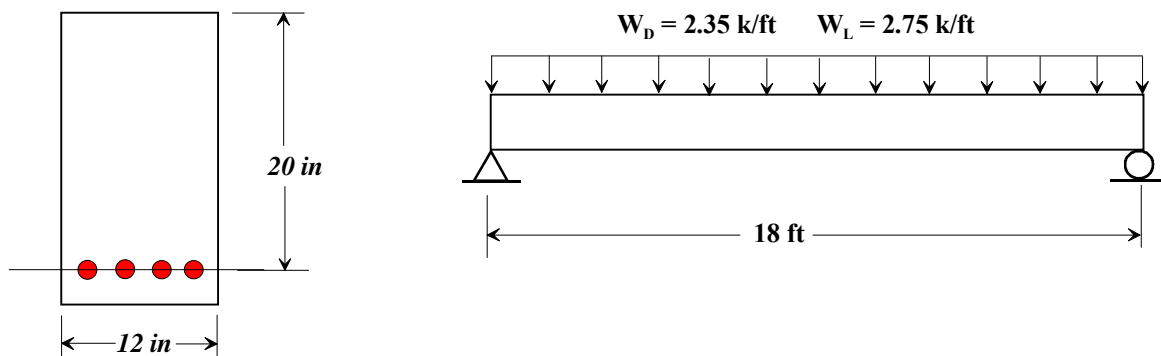
S = snow loads

W = wind loads

Note the new load factors for dead loads (1.2 new, 1.4 old), and other cases (1.6 new, 1.7 old).

Example 1 – Critical section for shear

A rectangular beam has a clear span of 18 ft. long, is 12 in. wide with an effective depth of 20 in. It supports a uniformly distributed dead load of 2.35 kips/ft and live load of 2.75 kips/ft. Determine the design factored shear at the critical section and compute the nominal shear strength of the concrete V_c . The specified compressive strength of the concrete $f'_c = 4,000$ psi.



Solution:

a) Factored Design Loads

$$W_u = 1.2 W_d + 1.6 W_l$$

$$W_u = 1.2(2.35) + 1.6(2.75) = 7.22 \text{ k/ft}$$

- b) Since the loading is symmetrical, the two reactions and the end shears are equal,

$$V_u = 7.22 \times 18 / 2 = 65 \text{ kips}$$

And the design shear at the critical section, a distance d from the face of the support is

$$V_u = 65 - (7.22 \times 1.67) = 53 \text{ kips}$$

- c) The nominal shear strength of the concrete is computed by equation 5:

$$V_c = 2 \sqrt{f'_c} b_w d$$

$$V_c = \frac{2 \times \sqrt{4,000} \times 12 \times 20}{1000} = 30.36 \text{ kips}$$

and $\phi V_c = 0.75 \times 30.36 = 22.77 \text{ kips}$

Since $V_u = 53 \text{ kips} > \phi V_c = 22.77 \text{ kips}$, web reinforcement is required in this beam.

Example 2 – Design of stirrups

For the beam in Example 1 above, determine the distance from the support through which stirrups are required, and find the required spacing of stirrups at the maximum critical section for shear. Stirrups are Grade 60 steel ($f_y = 60,000$ psi).

Solution:

- a) Find the distance from the critical section at which stirrups theoretically are no longer required:

$V_c = 30.36$ kips from Example 1

$$V_s = (V_u/\phi) - V_c$$

$$V_s = (53/0.75) - 30.36 = 40.31 \text{ kips}$$

From Figure 10, distance “a” is determined,

$$a = \frac{L}{2} \left[\frac{V_u - \phi V_c}{V_u} \right] = 9 \times \frac{65 - 22.77}{65} = 5.84 \text{ ft}$$

Note that distance “a” applies only to beams with uniformly distributed loads and whose reactions are equal in magnitude.

Furthermore, a minimum amount of web reinforcement is required whenever the factored shear force exceeds $\phi V_c/2$ or **11.4 kips** in this case. Again the distance from the face of the support to where minimum reinforcement must be applied is found from Fig. 10,

$$a' = \frac{L}{2} \left[\frac{V_u - \phi V_c/2}{V_u} \right] = 9 \times \frac{65 - 11.4}{65} = 7.42 \text{ ft}$$

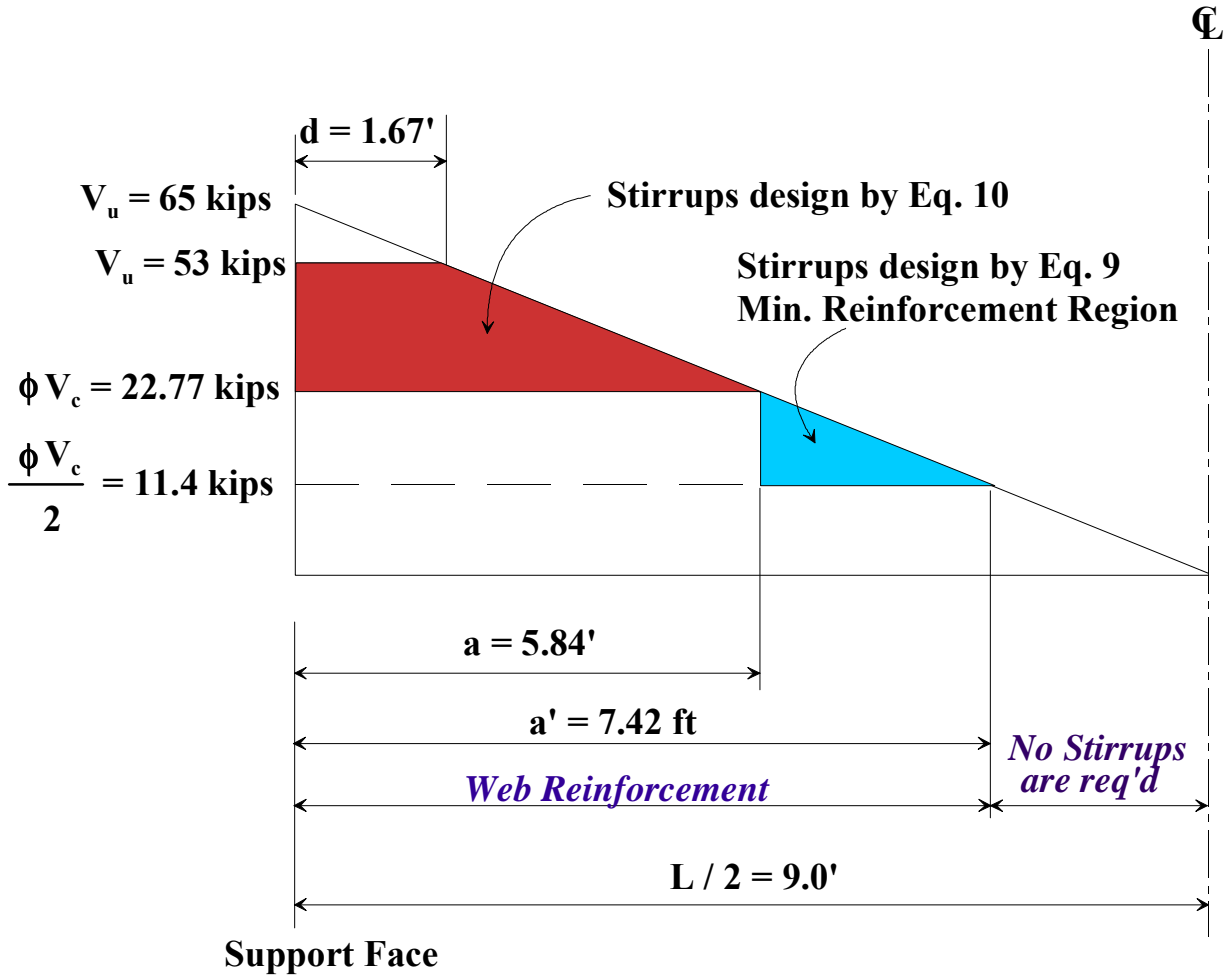


Figure No. 10
Shear Diagram

b) The spacing of stirrups at the critical section for shear is computed at $d = 1.67'$ from the face of support,

Assume #3 bars U-Stirrups, $A_v = 2 \times 0.11 = 0.22 \text{ in}^2$,

From Equation 3

$$V_s = (V_u / \phi) - V_c$$

$$V_s = (53/0.75) - 30.36 = 40.31 \text{ kips}$$

$$\text{Since } V_s \leq 4 \sqrt{f'_c} b_w d = 4 \sqrt{4,000} \frac{12 \times 20}{1000} = 60.72 \text{ kips}$$

The maximum stirrup spacing is limited to $d/2 = 10$ in. (See Table 2).

From Equation 10 and rearranging terms, the required stirrup spacing at the critical section is computed as:

$$s = \frac{A_v f_y d}{V_s} = \frac{0.22 \times 60 \times 20}{40.31} = 6.54 \text{ in}$$

Using a stirrup spacing of 6 1/2 in. throughout zone a = 5.84 ft., shown in Fig. 10, will lead to a conservative design. Beams of constant cross section and loaded with uniformly distributed load have shear forces that decrease in a straight line from a maximum value at the supports to the section of zero shear. Beyond distance $a' = 7.42$ ft. from the support to the midspan, no stirrups are required.

And finally the spacing required to meet the minimum web reinforcement is obtained from Equation 9,

$$A_v = 0.75 \sqrt{f'_c} \frac{b_w s}{f_y} \geq \frac{50 b_w s}{f_y} \quad (\text{Equation 9})$$

$$s = \frac{A_v f_y}{0.75 \sqrt{f'_c} b_w} = \frac{0.22 \times 60000}{0.75 \sqrt{4,000} 12} = 23.2 \text{ in.}$$

But not less than:

$$s = \frac{A_v f_y}{50 b_w} = \frac{0.22 \times 60000}{50 \times 12} = 22 \text{ in.}$$

Concluding, at the minimum reinforcement region, the required maximum stirrup spacing is limited to $s_{max} = d/2 = 10$ in. It is common practice to place the first stirrup at $s/2$ distance from the face of the support, or 5 in. for this case.

Example 3 – Beam without web reinforcement

A rectangular beam is to carry a shear force V_u of 40 kips. No web reinforcement is to be used, and $f'_c = 4,000$ psi. Find the minimum cross section as governed by shear.

Solution:

Since no web reinforcement is to be used, the cross-sectional dimensions must be chosen so that the applied shear V_u is no larger than one-half the design shear strength ϕV_c .

$$\text{Thus, } V_u = \phi \frac{1}{2} \left[2 \sqrt{f'_c} b_w d \right]$$

$$\text{Rearranging terms: } b_w d = \frac{2 V_u}{2 \phi \sqrt{f'_c}}$$

Therefore:

$$b_w d = \frac{40,000}{0.75 \sqrt{4,000}} = 843 \text{ in}^2$$

A beam with $b_w = 24$ in and $d = 35$ in will satisfy the design criteria of no web reinforcement.

Alternatively, if the minimum amount of web reinforcement given by Eq. 9 is used, the concrete shear resistance may be taken to the full value ϕV_c , and the beam cross-sectional dimensions could be reduced to $b_w = 16$ in and $d = 26.5$ in.

Example 4 – Web Reinforcement for Girder

A simply supported rectangular girder 14 in. wide having an effective depth of 24 in. has equal concentrated loads at the third points of a clear span of 24 ft. as shown below. The specified compressive strength of the concrete is 5,000 psi. Using a uniform load of 3,500 lb per linear foot for the girder weight, investigate the necessity of web reinforcement.

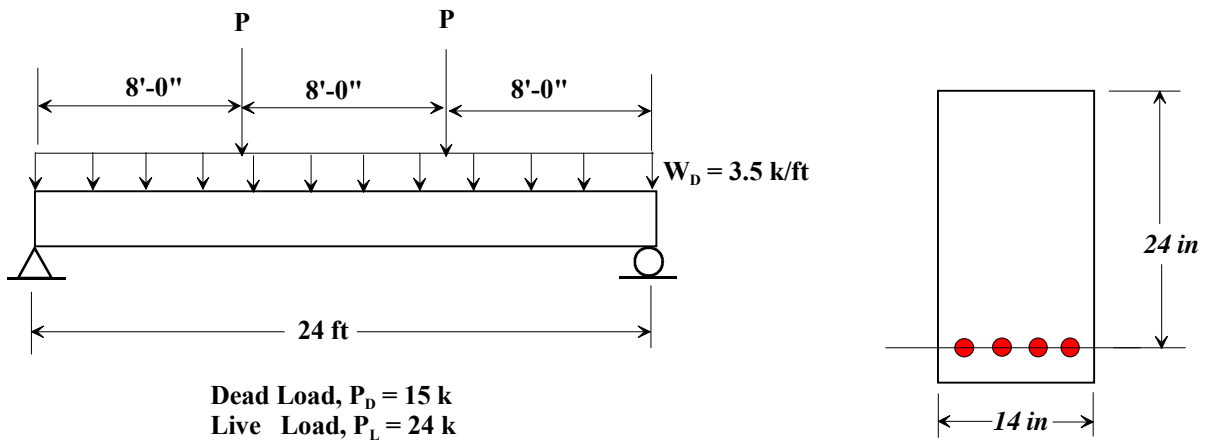


Figure No. 11

Solution:

- a) Compute the design loads,

$$\text{Concentrated Loads, } P_u = 1.2(15) + 1.6(24) = 56.4 \text{ kips}$$

$$\text{Uniform loads, } w_u = 1.2(3.5) = 4.2 \text{ kips/ft}$$

- b) The loading is symmetrical, therefore the design end shear is equal to one of the concentrated loads plus one-half the uniform load over the span, or

$$V_u = 56.4 + (4.2 \times 24/2) = 106.8 \text{ kips}$$

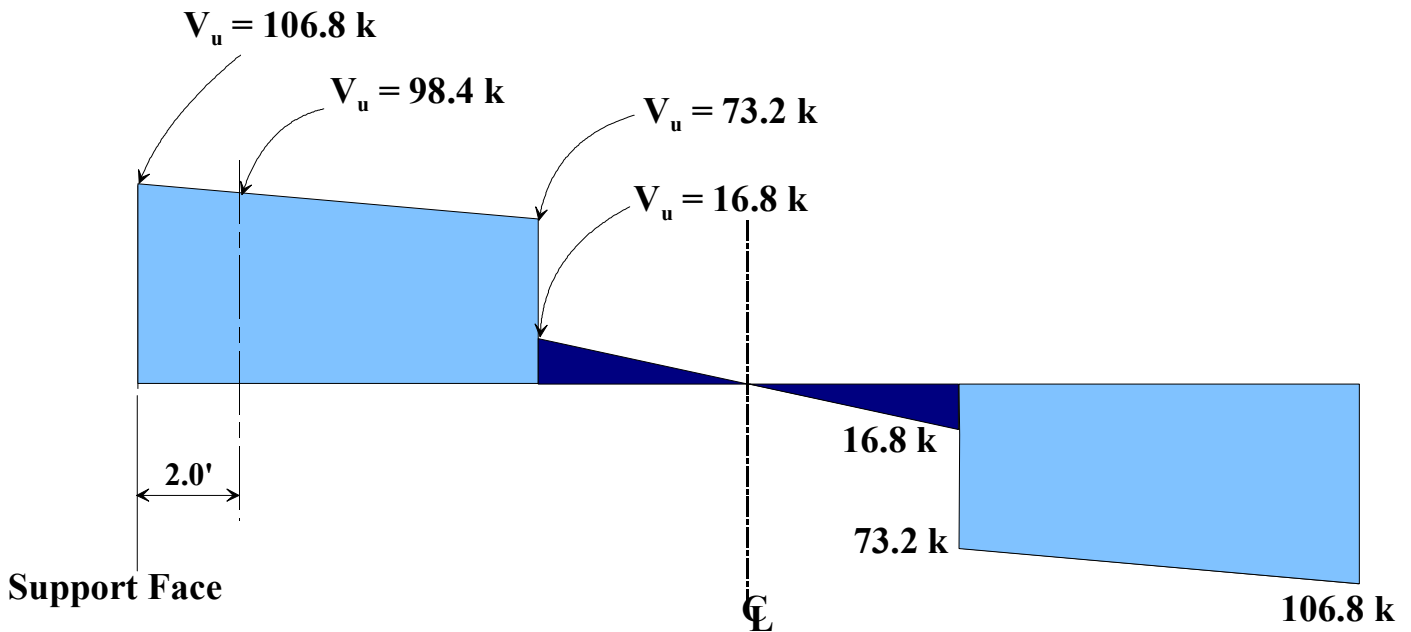
c) Compute the design shear at distance $d = 2$ ft. from the left support, and just to the left and right of P_u located 8 ft. from the support:

$$V_{u(2.0)} = 106.8 - (4.2 \times 2) = 98.4 \text{ kips}$$

$$V_{u(8-)} = 106.8 - (4.2 \times 8) = 73.2 \text{ kips}$$

$$V_{u(8+)} = 106.8 - 56.4 - (4.2 \times 8) = 16.8 \text{ kips}$$

d) Draw the shear diagram:



SHEAR FORCE DIAGRAM

The nominal concrete shear strength is computed as:

$$V_c = \frac{2 \sqrt{5,000} \times 14 \times 24}{1000} = 47.52 \text{ kips}$$

and $\phi V_c = 35.64 \text{ kips}$

Looking at the shear diagram, stirrups are required between the support and the concentrated loads since,

$$\phi V_c = 35.64 \text{ k} < V_u = 73.2 \text{ k}$$

At the portion of the girder between the two concentrated loads, we find that

$$V_u = 16.8 \text{ k} < \phi V_c / 2 = 17.82 \text{ k}$$

and no stirrups are required. Nevertheless, many structural engineers and designers prefer to provide stirrups at maximum spacing in this region (they also provide support to the main longitudinal reinforcement).

d) Design of Stirrups

The spacing of the stirrups will be uniform throughout the end thirds of the span and it is computed as:

Using #4 vertical stirrups Grade 60 steel ($f_y = 60 \text{ ksi}$): $A_v = 0.40 \text{ in}^2$

The design strength to be provided by the stirrups V_s , from Equation 3

$$V_s = (V_u / \phi) - V_c$$

$$V_s = (98.4 / 0.75) - 47.52 = 83.68 \text{ kips}$$

Since V_s is less than:

$$V_s \leq 4 \sqrt{f'_c} b_w d = 4 \sqrt{5,000} \frac{14 \times 24}{1000} = 95.04 \text{ kips}$$

The maximum spacing can be taken as $s_{\max} = d/2 = 12 \text{ in.}$

The calculated spacing of #4 vertical stirrups for the end thirds of the span is:

$$s = \frac{A_v f_y d}{V_s} = \frac{0.40 \times 60 \times 24}{83.62} = 6.88 \text{ in}$$

Therefore, provide #4 stirrups @ 6 1/2" on center for the two end thirds of the girder span, and #4 @ 12" on center for the center span between the concentrated loads.

Conclusion:

This course has covered the basic principles related to the design of shear reinforcement and requirement for nonprestressed reinforced concrete rectangular beams using the latest edition of the Building Code Requirements for Structural Concrete ACI 318-02.

The items discussed in this lecture included the principle of shear strength as it relates to diagonal tension cracks, evaluation of shear strength for concrete beams and the lower and upper bound of shear strength requirement, design of shear reinforcement, critical sections for shear reinforcement, and the new ACI 318 code provisions for shear strength of nonprestressed concrete beams.

References:

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