Compressible Flow Modeling In a Constant Area Pipe

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Course Content

Figure 1 shows a system that is designed to power a gearbox through the use of a turbine by expansion of a constant mass flow rate helium gas. In this system the designer wants to control the mass flow rate using a regulator and a sonic orifice. It is imperative that the orifice controls the mass flow to achieve the required turbine output for the gearbox.

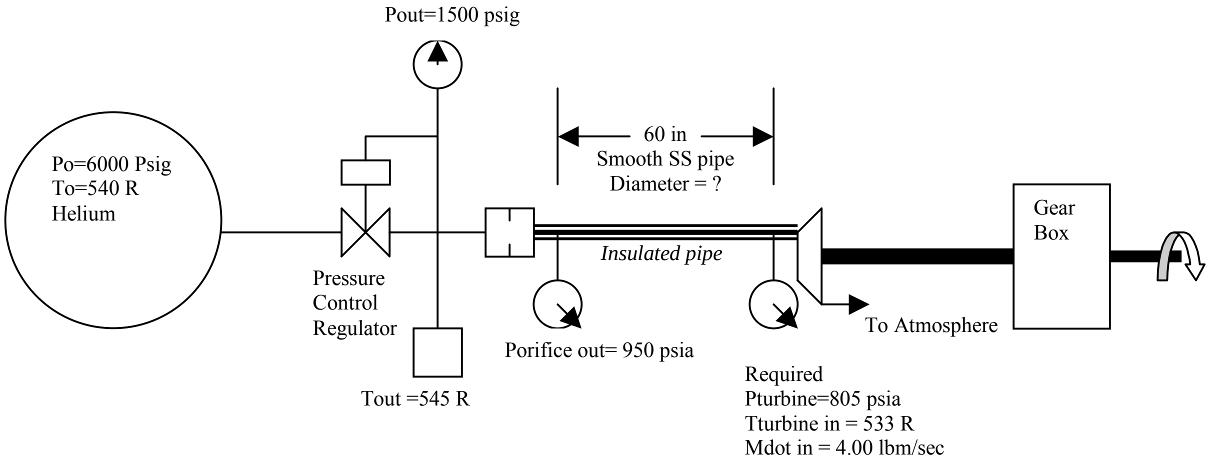

Figure 1: Example Compressible Flow System

The required turbine input conditions are given at the inlet of the turbine. The job of the engineer is to properly size the pipe diameter to achieve the turbine inlet pressure and temperature of the helium gas. The regulator has been set to maintain an inlet orifice pressure of 1500 psia and the orifice has been sized to choke and control the mass flow rate at 4.00 lbm/sec. The maximum backpressure from the downstream system resistance is 1050 psia. Any pressure above that will unchoke the orifice and reduce the mass flow rate. To stay conservative, the engineer's goal will be to achieve a backpressure or orifice out recovery pressure of no greater than 950 psia.

The following instructions will review compressible flow properties of a gas assuming the flow will be adiabatic and with frictional losses (Fanno Flow). In addition, a Fanno Flow algorithm will be derived to determine the required pipe diameter for the system described in Figure 1.

Adiabatic Flow with Friction In a Constant Area Pipe

Friction cannot be ignored in a long constant area pipe. In addition, if the pipe is insulated or the velocities of the gases are high enough that gas temperature is not effected by heat transfer from or to the surroundings, then modeling can be performed assuming adiabatic flow. Adiabatic flow cannot be assumed in long unsulated pipes in which the velocity of the gas is slow and the gas temperature will be affected by the heat transfer.

The following Figure 2 and Table 1 summarizes the effects of Fanno flow properties in a constant area pipe. The reader should review Figure 2 and Table 1 until he/she has a complete understanding. It is imperative at this point to understand static pressure, total pressure, and temperature always decreases when velocity of a gas entering a pipe is subsonic, and therefore the density decreases. Because the density of the gas is decreasing then the velocity of the gas particles must increase to maintain a constant flow rate. If the velocity of the gas becomes equal to the speed of sound, then the flow rate will choke at the end of the pipe, and the pipe can take control of the total flow rate by unchoking the flow-controlling orifice in the case of Figure 1 system. Thus the operator of the system will lose the capability of knowing the exact flow rate based on the orifice upstream pressure, temperature, and orifice Cd.

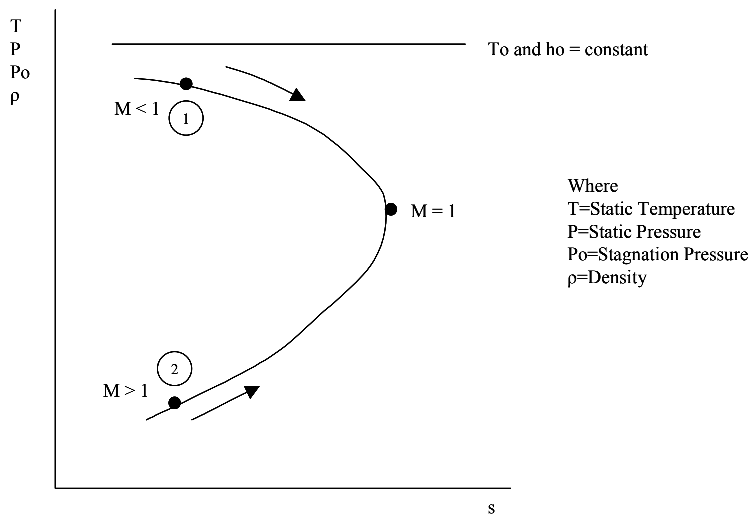

Figure 2: Schematic Ts Diagram Fanno Flow Line for a Constant Area Duct

Property	Duct Entry Subsonic M<1	Duct Entry Supersonic M>1	Obtained from:
Stagnation Temperature, To	Constant	Constant	Energy Equation
Entropy, s	Increases	Increases	Second Law
Stagnation Pressure, Po	Decreases	Increases	Entropy increases
Temperature, T	Decreases	Increases	Shape of Fanno line
Static Pressure, P	Decreases	Increases	Equation of state, and effects on ρ , T
Density, ρ	Decreases	Increases	Continuity equation and effect on V
Velocity, V	Increases	Decreases	Energy Equation and trend of T
Mach Number, M	Increases	Decreases	Trends of V, T, and definition of M

Table 1: Summary of Effects of Friction on a Compressible Gas Flowing In a Constant Area Pipe (Table copied from Introduction to Fluid Mechanics by Fox & McDonald 4th Edition)

Figure 2 also shows what happens if the velocity of the gas entering a pipe is supersonic. Friction will cause the pressure and temperature to increase in the direction of flow. The density will increase and the velocity will decrease to maintain the constant mass flow rate. Eventually the velocity in the pipe becomes sonic. In this course, the discussion and quiz will refer to modeling compressible flow with the velocity of the gas entering a constant area pipe at subsonic conditions.

Figure 3 is a diagram of gas flowing in a constant area pipe. The key to predicting gas properties using FANNO flow is knowing either the inlet or the end condition pressure, temperature, and mass flow rate of the gas. In the case of this course, the engineer knows the pipe end conditions.

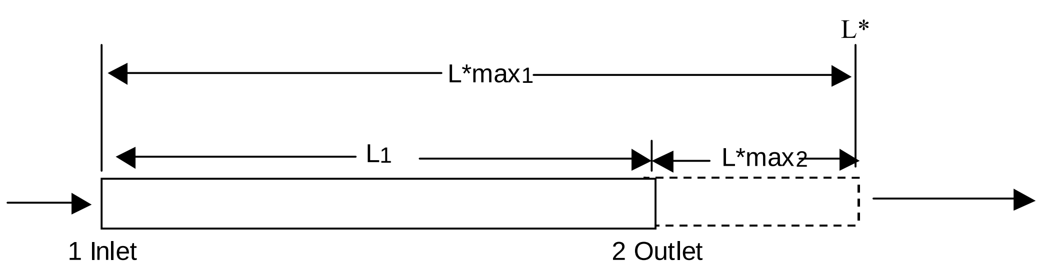

Figure 3: Constant Area Pipe

Reviewing Figure 3, one can see that L* is some where in space downstream (hopefully) of the outlet of the pipe. L* is a point where the flow velocity will be predicted to reach the speed of sound and the mach number will be one (sonic). If L*max 1 or 2 is calculated or predicted to be a length less than L_1 (pipe length), then the sonic point is inside the pipe and the flow will become sonic at the exit. Typically this is called Fanno choking. As explained in the preceding text, once the pipe becomes choked it could possibly take control of the mass flow rate. It is not uncommon to have more than one choking points in a system, but a problem can occur if the pipe choking creates enough resistance to unchoke the upstream flow-controlling device. If the upstream flow-controlling device is instrumented to measure upstream pressure and temperature, then the pressure can be regulated to change the mass flow rate using energy equations. Unchoking such a device due to excessive resistance negates the upstream flow rate predicting capability.

Using the Figure 3 as a reference and the equations described in the following algorithm, one can easily solve for conditions at point 1 or point 2 by simply calculating the mach number at either point using the L*max1 or L*max2 respectfully. The remainder of the text will provide step-by-step directions to demonstrate this logic using the described problem in Figure 1.

Solving for the Diameter

Step 1: Since the conditions are known for point 2 of Figure 3, then the analysis will start by solving for the Mach number. However before deriving conditions at point 2, an initial guess of the pipe diameter is required. Several guesses may need to be iterated through the algorithm before achieving the proper pressure drop. For the first guess, the author will use a nominal pipe diameter of 1.25 inch for a seamless stainless steel pipe with a wall thickness of 0.120 inches and an inside diameter of 1.01 inch.

$$A = \frac{\pi D_i^2}{4} = 0.7853982 D_i^2 \quad (1)$$

$$\rho = \frac{p}{ZRT} \quad (2)$$

where $Z = 1$ if the gas is assumed Ideal. However for this exercise the author chose to treat the problem as Real Gas and Z was found using NIST.

$$velocity = \frac{\overset{\circ}{m}}{\rho A} = 183.3465 \frac{\overset{\circ}{m}}{\rho D_i^2} \quad (3)$$

$$\overset{\circ}{V} = \frac{\overset{\circ}{m} * 60}{\rho_{std}} \quad (4)$$

$$Mach\# = \frac{v}{\sqrt{kRg_cT}} = \frac{0.1762268v}{\sqrt{kRT}} \quad (5) \text{ where k is the ratio of specfic heats } \frac{c_p}{c_v}$$

Known Outlet Conditions 2		
INPUT		
P2=	805	psia
T2=	533	R
mdot=	4.000000	lbm/s
L1=	60	in
Dh=	1.01	in
Gas Type =	8	See Info on GASP Functior
Calculated Inlet Conditions 2		
Gas=	Helium	
Gas Constant=	386.040	(ft-lbf)/(lbm-R)
density2=	0.545266	lbm/ft3
Ai=	0.801	in2
vdot=	22776.092	scfm
velocity2=	1318.504	ft/s
k =	1.667	specific heats ratio
Mach 2 =	0.3967	

Table 2: Input at Point 2 and Calculated Conditions (reference Figure 1)

Step 2: Once the mach number is known at point 2, the following conditions can be determined at the imaginary sonic point L* of Figure 3 using the following algorithm:

$$\mathrm{Re} = \frac{\rho v D}{\mu} \quad (6)$$

where $\mu = 0.000012$ lbm/ft-sec for helium

$$f = \left[1.14 - 2\log_{\log}\left(\frac{\varepsilon}{D_h} + \frac{21.25}{\mathrm{Re}^{0.9}} \right) \right]^{-2} \quad (7)$$

where $\varepsilon = 0.000984$ for seamless smooth stainless steel duct

$$T^* = \frac{(T)(2)[1+(0.5)(k-1)M_i^2]}{1+k} \quad (8)$$

$$p^* = \frac{(P_i)(M_i)}{\sqrt{\dfrac{1+k}{2[1+(0.5)(k-1)M_i^2]}}} \quad (9)$$

$$\frac{fL^*\max_2}{D_h} = \left[\left(\frac{1-M_2^{\,2}}{kM_2^{\,2}} \right) + \left(\frac{1+k}{2k} \right) \ln\left(\frac{(1+k)M_2^{\,2}}{2[1+(0.5)(k-1)M_2^2]} \right) \right] \quad (10)$$

$$L^*\max_2 = \left(\frac{D_h}{f} \right) \left[\left(\frac{1-M^2}{kM^2} \right) + \left(\frac{1+k}{2k} \right) \ln\left(\frac{(1+k)M^2}{2[1+(0.5)(k-1)M^2]} \right) \right] \quad (11)$$

Table 3 gives the derived results of the conditions at L*max2. Note that L*max2 is positive showing the sonic point is downstream of the pipe. Having this data enables us to determine L*max1 which will be explained in step 3.

T*=	**420.682**	R
P*=	**283.736**	psia
Darcy friction factor f=	**0.019599031**	
L*max2=	**99.021**	in
Line Length Check	**Not Choked**	
fL2/Dh=	**1.164**	
fL2*/Dh=	**1.921**	

Table 3: Calculated Conditions at Imaginary Sonic Point L*max2

Step 3: The following step derives L*max1 by simply adding the length of the pipe to L*max2 derived in step 2.

$$L^* \max_1 = L_1 + L^* \max_2 \quad (12)$$

Step 4: Once L*max1 is calculated the following equation is used to derive the Mach number at point 1. This process can be simplified greatly by using some type of solver program or a spreadsheet. In table 4, the Mach number is derived by iterating a mach number until the difference between the calculated L*max1 and iterated L*max1 is equal to zero within six significant digits.

$$L^* \max_1 = \left(\frac{D_h}{f}\right)\left[\left(\frac{1-M_1^{\,2}}{kM_1^{\,2}}\right)+\left(\frac{1+k}{2k}\right)\ln\left(\frac{(1+k)M_1^{\,2}}{2[1+(0.5)(k-1)M_1^2]}\right)\right] \quad (13)$$

Calculated Inlet Conditions 1		
Calculated L1 + L*max 2 = L*max1	159.021	in
Guess Mach 1 =	0.3391	Use Solver
Iterated L*max1=	159.021	in
Difference between Calculted and Iterated 4fl2*/Dh =	0.000000	Iterates to 0000

Table 4: Calculated Mach number at Point 1 Pipe Inlet

Step 5: Now that the Mach number is known at the inlet of the pipe, the conditions at the inlet can be derived. Most importantly, the pressure and pressure drop can be determined using a 1.25 in diameter tubing.

$$P_1 = \frac{p^*}{M_1}\sqrt{\frac{1+k}{2[1+(0.5)(k-1)M_1^2]}} \quad (14)$$

$$T_1 = \frac{T^*(1+k)}{2[1+(0.5)(k-1)M_1^2]} \quad (15)$$

P1=	948.105	psia
T1=	540.256	R

Table 5: Pipe Inlet Conditions

Conditions shown at Table 5 show that using a 1.25 inch stainless steel tubing with an inside diameter of 1.01 inch provides a downstream orifice recovery pressure of 948 psia. Our goal was a maximum pressure of 950 psia. That is 0.20% difference in pressure, and well within operational requirements for this sample problem.

Course Summary

Fanno flow modeling is used extensively in one-dimensional modeling of gases when the Mach number is greater than 0.3, the process is assumed adiabatic, and pressure loss due friction is considered. However, it not uncommon to use Fanno flow modeling for velocities lowered than Mach 0.3 as long as the system as long as the same assumptions are applied. Installing this algorithm into program or a spreadsheet reduces the workload greatly when iterating several different pipe geometry configurations. In addition, alternating inlet conditions can lead to a quick understanding of the effect of pressure and temperature on the gas density and gas velocity which in turn effects choking the flow inside the pipe or choking the flow outside of the pipe at some imaginary point L*. Once that understanding is understood, then designing or analyzing of compressible systems is simplified greatly.

Reference:

1. Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 4th Edition

2. Philip Hill and Carl Peterson, Mechanics and Thermodynamics of Propulsion, 2nd Edition

3. Michael R. Lindeburg PE, Mechanical Engineering Reference Manual for PE Exam, 11th Edition

4. Robert D. Blevins, Applied Fluid Dynamics Handbook, Reprint Edition 1992